AGNOSTIC TESTS OF GRAVITY

TESSA BAKER

Oxford Uni. & U.Penn

FULBRIGHT COMMISSION

OUTLINE

1. A map of alternative gravity.
2. A new, agnostic framework for testing gravity.
3. Constraints, current and future.

1. A GUIDE TO ALTERNATIVE GRAVITY.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{M_{Pl}^2}{2} \int \sqrt{-g}\, d^4x \left[\, R \,\right]$$

Five options:

LOVELOCK'S THEOREM (1971)

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$$S_{\rm grav} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}\, d^4x \left[\phi R - \frac{\omega(\phi)}{\phi}\left(\nabla\phi\right)^2 - 2V(\phi)\right]$$

Five options:

1. Add new field content.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\text{grav}} = \frac{{M_D}^2}{2} \int \sqrt{-\gamma}\, d^D x \left[\mathcal{R} + \alpha\, \mathcal{G} \quad \right]$$

Five options:

1. Add new field content.
2. Higher dimensions.

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$$S_{\rm grav} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}\, d^4x \Big[R + \beta_1 R \nabla_\mu \nabla^\mu R + \beta_2 \nabla_\mu R_{\beta\gamma} \nabla^\mu R^{\beta\gamma}\Big]$$

Five options:

1. Add new field content.
2. Higher dimensions.
3. > 2nd order derivatives in the field equations.

LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

$$S_{\mathrm{grav}} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}\, d^4x \left[R + f\left(\frac{1}{\Box}R\right).\right]$$

Five options:

1. Add new field content.
2. Higher dimensions.
3. > 2nd order derivatives in the field equations.
4. Non-locality.

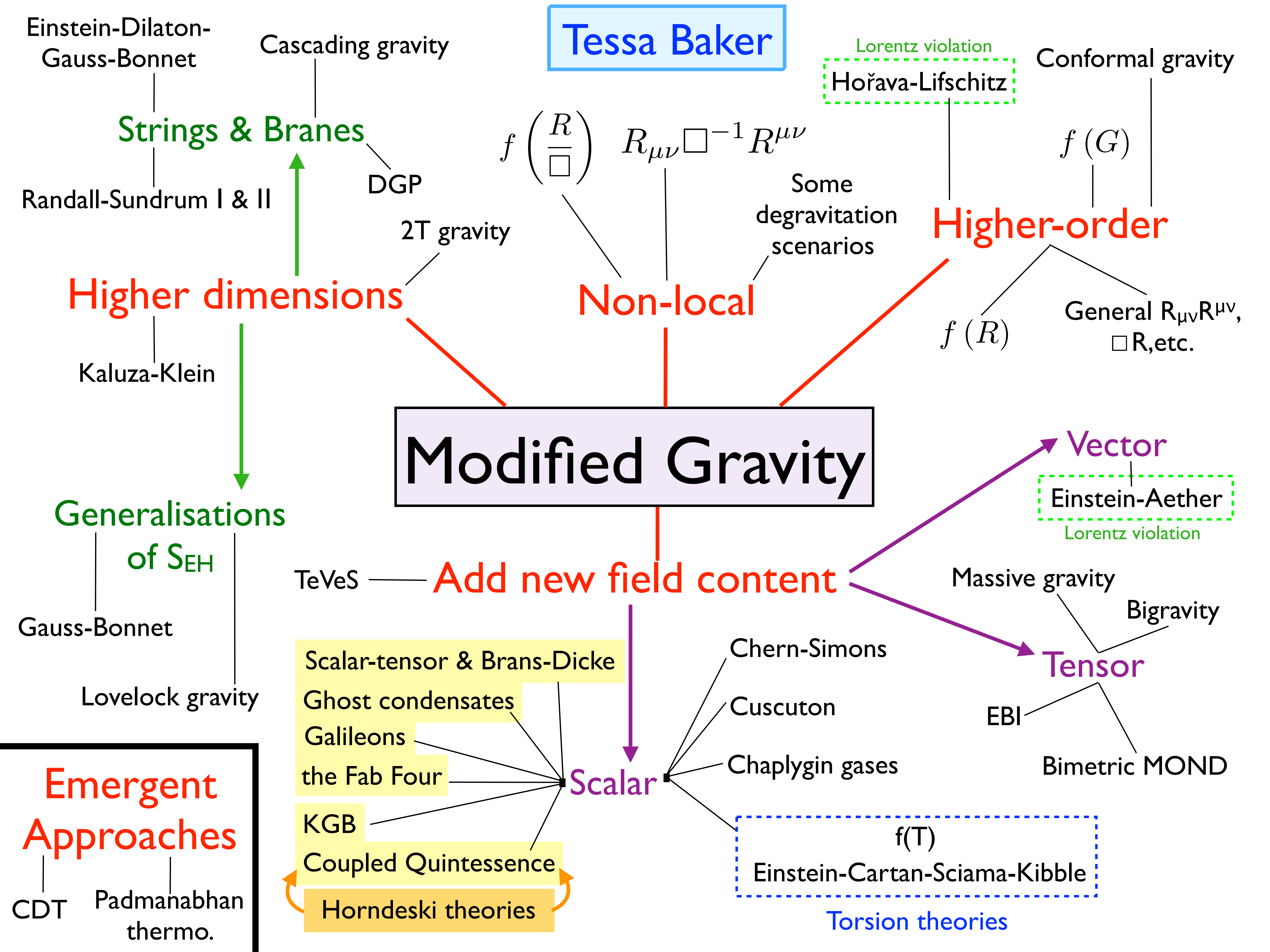
LOVELOCK'S THEOREM (1971)

"The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

? ? ? ? ? ? ?

Five options:

1. Add new field content.
2. Higher dimensions.
3. > 2nd order derivatives in the field equations.
4. Non-locality.
5. Radically change our action principle (emergence).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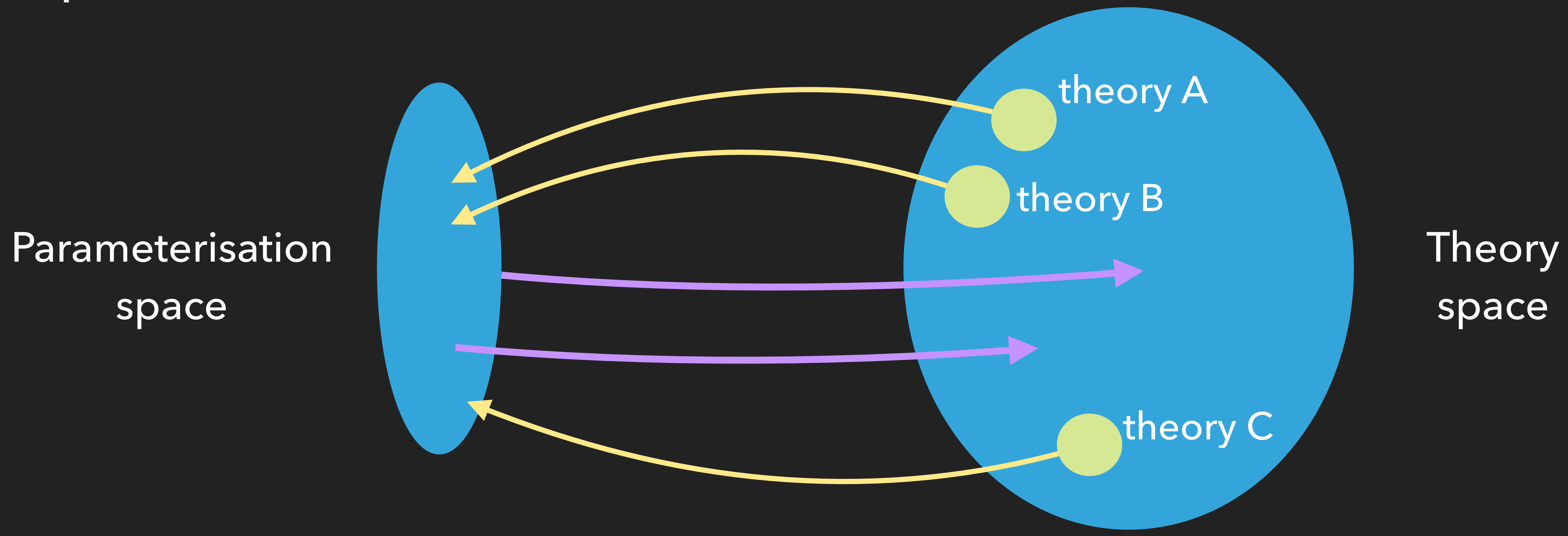
2. PARAMETERISED FRAMEWORKS FOR GRAVITY.

AGNOSTIC PARAMETERISATIONS

- Different theories ⇒ different specifications of the framework's parameters.



- Test a model-independent framework, à la PPN (Parameterised Post-Newtonian…).
- Downside: the framework `parameters' are really functions of time…and sometimes scale, too.

AGNOSTIC PARAMETERISATIONS

Specify the field content and symmetries.

Construct most general quadratic action from allowed building blocks.

Apply gauge symmetries $\Rightarrow$ fixes relationships between new terms.

Maximal set of `parameters' to be constrained.

STEP 1

Specify the field content and symmetries.

- Do we want to add new scalar, vector or tensor fields to gravity?

E.g. Horndeski Einstein-Aether bigravity

- Isotropic + homogeneous cosmological background.

$$\Rightarrow \quad a(t)\,, \quad H(t)$$

- Linear diffeomorphism invariance.

i.e. GR's lack of a preferred coordinate system.

AGNOSTIC PARAMETERISATIONS

Specify the field content and symmetries.

↓

Construct most general quadratic action from allowed building blocks.

STEP 2

Construct most general quadratic action from allowed building blocks.

- Use a '3+1 split' or 'ADM decomposition'.

  ⇒ splits spacetime into a timelike normal + spacelike surfaces.

$$g_{\mu\nu} = \underbrace{-n_\mu n_\nu}_{\text{timelike normal}} + \underbrace{h_{\mu\nu}}_{\text{3D spatial slices}}$$

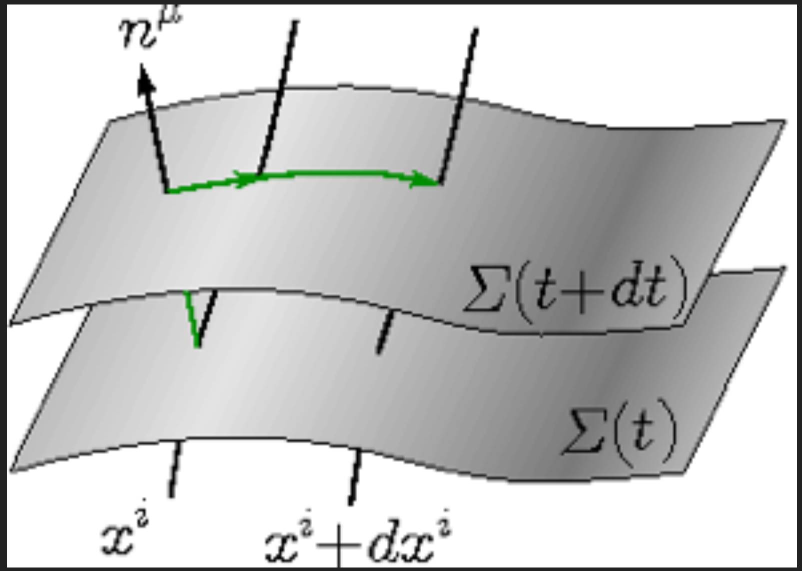


Image: Yi Wang.

STEP 2

Construct most general quadratic action from allowed building blocks.

- The ADM formalism hands us a set of objects that make up the 4D metric.

$\Rightarrow$ Building blocks:

$$N,\ N^i,\ h_{ij},\ K^i_j,\ R^i_j$$ describe the metric

$$+$$

$$\phi \ \text{ or } \ \tilde{\mathrm{V}} \ \text{ or } \ \mathrm{q}_{\mu\nu}$$ new fields (not present in GR)

$$+$$

$$\rho,\ \ P,\ \ v^i$$ usual fluid matter sector

STEP 2

Construct most general quadratic action from allowed building blocks.

- Put these building blocks into a vector (just convenient):

$$\vec{\Theta} = \left(N,\, N^i,\, h_{ij},\, K^i_j,\, R^i_j,\, \phi, \ldots\right)$$

- Taylor expand the gravitational Lagrangian:

$$L \simeq \bar{L} + \underbrace{L_{\Theta_a}}_{\downarrow\ \frac{\partial L_{\Theta_a}}{\partial \Theta_a}} \delta\Theta_a + \underbrace{\frac{1}{2} L_{\Theta_a \Theta_b} \delta\Theta_a \delta\Theta_b}_{\text{Gives us linearised grav. field equations.}} + \ldots$$

STEP 2

$$L \simeq \bar{L} + L_{\Theta_a}\,\delta\Theta_a + \frac{1}{2}\,L_{\Theta_a\Theta_b}\delta\Theta_a\delta\Theta_b + \ldots$$

Construct most general quadratic action from allowed building blocks.

$$\delta_2 S = \int dt\, d^3x\, N\sqrt{-h}\,\Big[\, L_{hR}(t)\,\delta h^i_j \delta R^j_i \;+\; L_{NK}(t)\,\delta N \delta K$$

$$+\; L_{h\,\partial N^i}(t)\,\delta h_{ij}\,\partial^i \delta \dot{N}^j + L_{\partial N \partial N}(t)\,\partial^i \delta N\,\partial_i \delta N \;+\; \ldots$$

$$+\; L_{\partial\phi\partial\phi}(t)\,\partial^i \delta\phi\,\partial_i \delta\phi \;+\; L_{\phi R}(t)\,\delta\phi\,\delta R \;+\; \cdots$$

$$+\; \text{usual fluid matter sector} \;\Big]$$

AGNOSTIC PARAMETERISATIONS

Specify the field content and symmetries.

$\downarrow$

Construct most general quadratic action from allowed building blocks.

$\downarrow$

Apply gauge symmetries $\Rightarrow$ fixes relationships between new terms.

STEP 3

Apply gauge symmetries ⇒ fixes relationships between new terms.

- Apply gauge transformation (linear diffeomrophism):

$$x^\mu \to x^\mu + \epsilon^\mu$$

$$\downarrow$$

$$\epsilon^\mu = \left(\pi,\ \partial^i \epsilon\right)$$

- Perturbed building blocks δN, δN^i, δh_{ij} etc. transform in known way.

E.g. $\delta N^2 \to \delta N^2 - 2\delta N \dot{\pi}$

STEP 3

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- Perturbed building blocks δN, δN^i, δh_{ij} etc. transform in known way.

$$\Rightarrow \quad \delta_2 S \to \delta_2 S + \underbrace{\left[\begin{array}{c}\text{terms linear in} \\ \delta N,\ \delta N^i,\ \delta h_{ij}\ \text{etc.}\end{array}\right] \times \left(\pi \ \text{ or } \ \epsilon\right)}_{\text{must vanish}}$$

STEP 3

$x^\mu \to x^\mu + (\pi, \partial^i \epsilon)$

Apply gauge symmetries ⇒ fixes relationships between new terms.

- i.e. the following must vanish:

$$\Big[\, (\ldots)\,\delta N + (\ldots)\,\partial_i \delta N^i + (\ldots)\,\partial^i\partial^j(\delta h_{ij}) + (\ldots)\,\delta\phi \,\Big]\,\pi$$

$$+$$

$$\Big[\, (\ldots)\,\delta N + (\ldots)\,\partial_i \delta N^i + (\ldots)\,\partial^i\partial^j(\delta h_{ij}) + (\ldots)\,\delta\phi \,\Big]\,\epsilon$$

Combinations of Taylor coefficients, $L_{hR}(t)$, $L_{\varphi N}(t)$ etc.
PLUS cosmological background: a(t), H(t).

FLASHBACK

$$L \simeq \bar{L} + L_{\Theta_a}\,\delta\Theta_a + \frac{1}{2}\,L_{\Theta_a\Theta_b}\delta\Theta_a\delta\Theta_b + \ldots$$

Construct most general quadratic action from allowed building blocks.

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$$+\; \text{usual fluid matter sector} \;\Big]$$

STEP 3

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Combinations of Taylor coefficients, $L_{hR}(t)$, $L_{\varphi N}(t)$ etc.
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STEP 3

$x^\mu \to x^\mu + (\pi,\, \partial^i \epsilon)$

Apply gauge symmetries ⇒ fixes relationships between new terms.

- Nondynamical symmetry ⇒ all $(\ldots)$ must vanish, always.

- Result: a set of **Noether constraints** linking $L_{hR}(t)$, $L_{\varphi N}(t)$ etc.

  E.g. $\dot{L}_{hR} + 3H L_{K\partial N} - 5H^2 L_{\phi R} = 0$

- Tedious-but-easy exercise in elimination of variables.

$$\Rightarrow \quad L_{K\partial N} = \ldots \text{ other coeffs} \qquad L_{\dot{N}\dot{N}} = 0$$

$$L_{\phi R} = \ldots \text{ other coeffs} \qquad L_{h\partial^2 N} = 0$$

FLASHBACK

$$L \simeq \bar{L} + L_{\Theta_a}\,\delta\Theta_a + \frac{1}{2}\,L_{\Theta_a\Theta_b}\delta\Theta_a\delta\Theta_b + \ldots$$

Construct most general quadratic action from allowed building blocks.

$$\delta_2 S = \int dt\, d^3x\, N\sqrt{-h}\,\Big[\, L_{hR}(t)\,\delta h^i_j \delta R^j_i \;+\; L_{NK}(t)\,\delta N \delta K$$

$$+\; L_{h\,\partial N^i}(t)\,\delta h_{ij}\,\partial^i \delta \dot{N}^j + L_{\partial N \partial N}(t)\,\partial^i \delta N\,\partial_i \delta N + \;\ldots$$

$$+\; L_{\partial\phi\partial\phi}(t)\,\partial^i\delta\phi\,\partial_i\delta\phi \;+ L_{\phi R}(t)\,\delta\phi\,\delta R \;+\; \cdots$$

$+$ usual fluid matter sector $\Big]$

AGNOSTIC PARAMETERISATIONS

Specify the field content and symmetries.

Construct most general quadratic action from allowed building blocks.

Apply gauge symmetries $\Rightarrow$ fixes relationships between new terms.

Maximal set of `parameters' to be constrained.

STEP 4

Maximal set of `parameters' to be constrained.

- Rewrite original action in terms of terms of non-redundant coeffs.

- Scalar field case:
  Initial ~ 40 unknown coeffs collapse to just **4** non-redundant ones.

$$\delta_2 S = \int d^3x\, dt\; a^3 \frac{M(t)^2}{2} \Big[R^{(4\mathrm{D})} + \alpha_T(t)\, \delta_2\Big(\sqrt{h} R / a^3\Big) + \alpha_K(t) H^2 \delta N^2 + 4\alpha_B(t) H \delta K \delta N \Big]$$

STEP 4

$\alpha_T(t)$: speed of gravitational waves, $c_T^2 = 1 + \alpha_T$.

$\alpha_K(t)$: kinetic term of scalar field.

$\alpha_B(t)$: `braiding' – mixing of scalar + metric kinetic terms.

$\alpha_M(t) = \dfrac{1}{H} \dfrac{d \ln M^2(t)}{dt}$: running of effective Planck mass.

[$\alpha_H(t)$: Optional: disformal transformations.

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} + \Gamma\, \partial_\mu \phi \partial_\nu \phi$$]

3. INTERFACE WITH OBSERVATIONS

WHY SHOULD YOU CARE?

- Maximal, model-independent description of modified gravity.

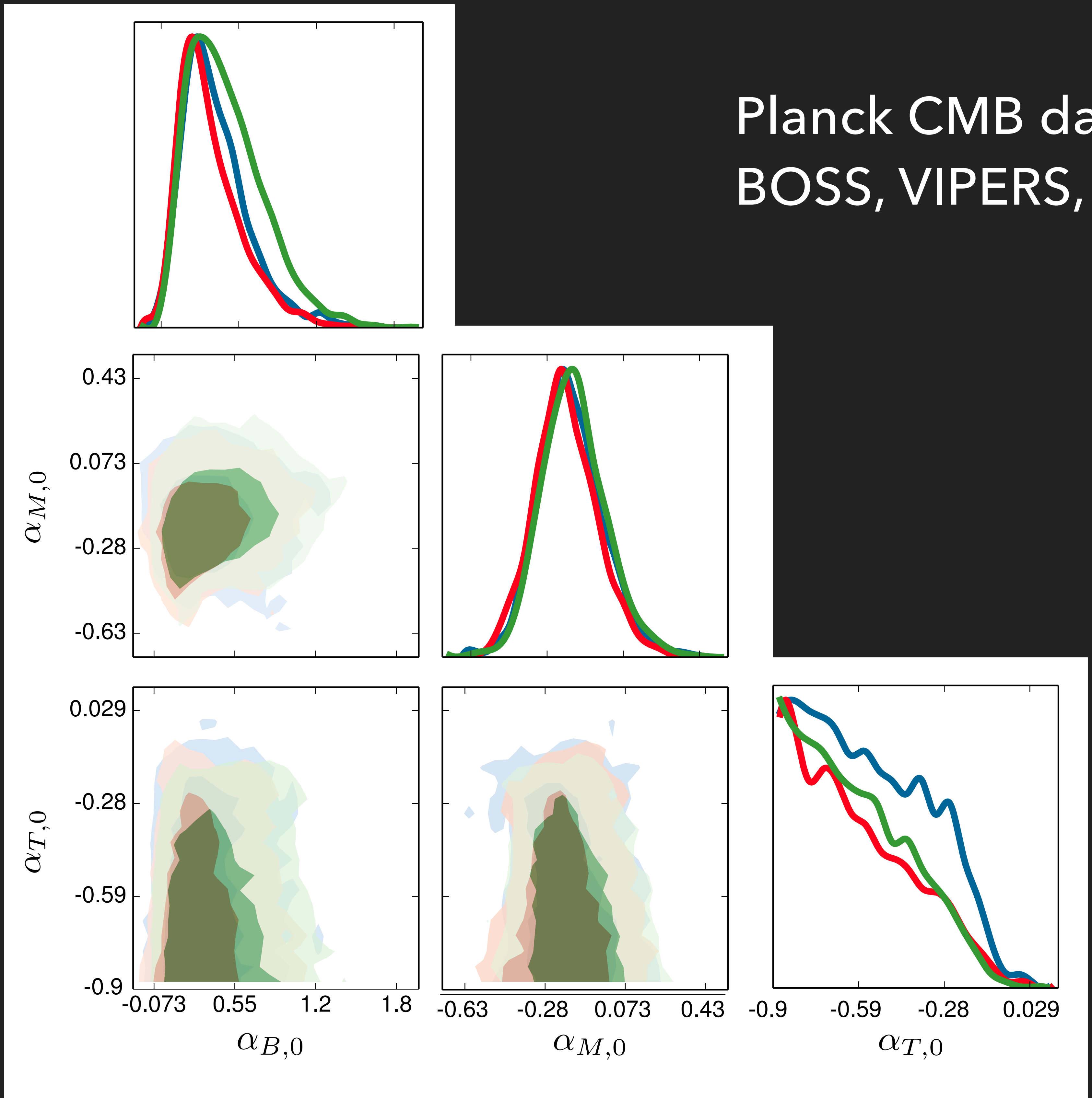
  ⇒ Your results are protected from changing theory fashions.

- This is the most general parameterisation available.

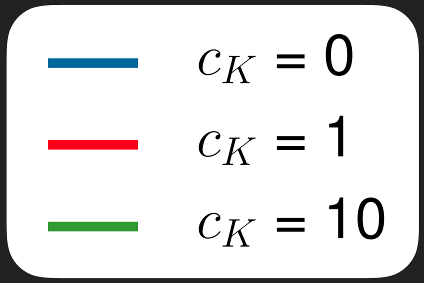
Fields	Functions	Theory family
$g_{\mu\nu}$	0	GR
$g_{\mu\nu},\ \phi$	5	Horndeski
$g_{\mu\nu},\ A^{\mu}$	10	Vector-tensor
$g_{\mu\nu},\ A^{\mu},\ \lambda$	4	Einstein-Aether
$g_{\mu\nu},\ q_{\mu\nu}$	? (in progress)	Bigravity

previous works

THE DATA IS COMING



Planck CMB data + galaxy surveys: BOSS, VIPERS, WiggleZ.

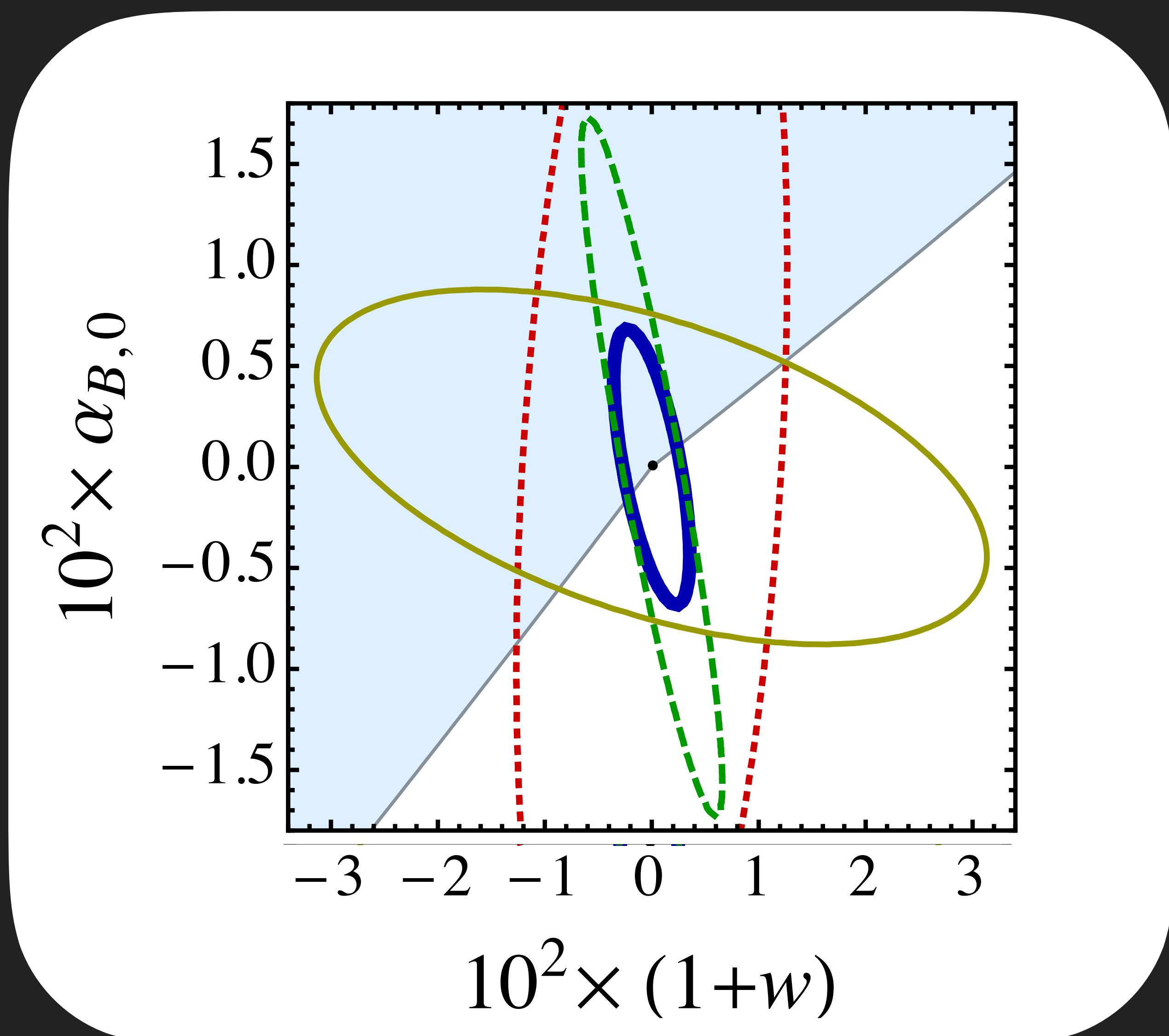


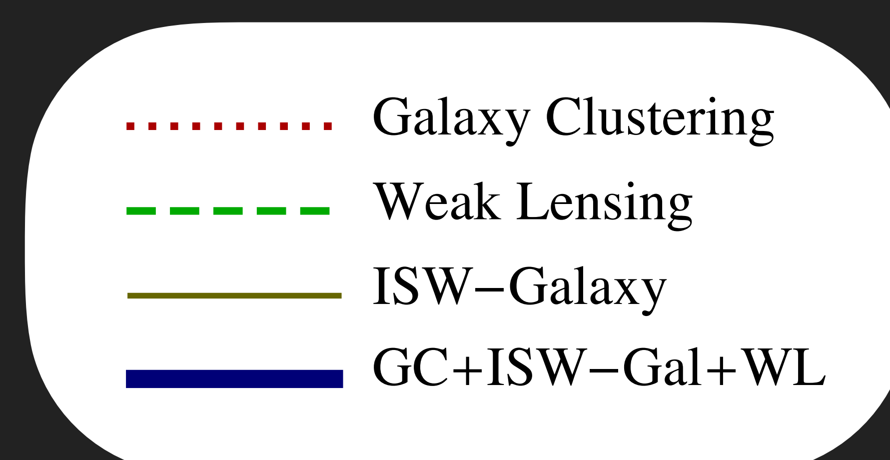
Bellini et al., 2015.

THE DATA IS COMING

Gleyzes et al., 2015.

- Example forecast for a Euclid-like survey:





- Potentially DES 3-year data?

FREEBIES

Lagos, TB, Ferreira & Noller (2016).

- We have code!

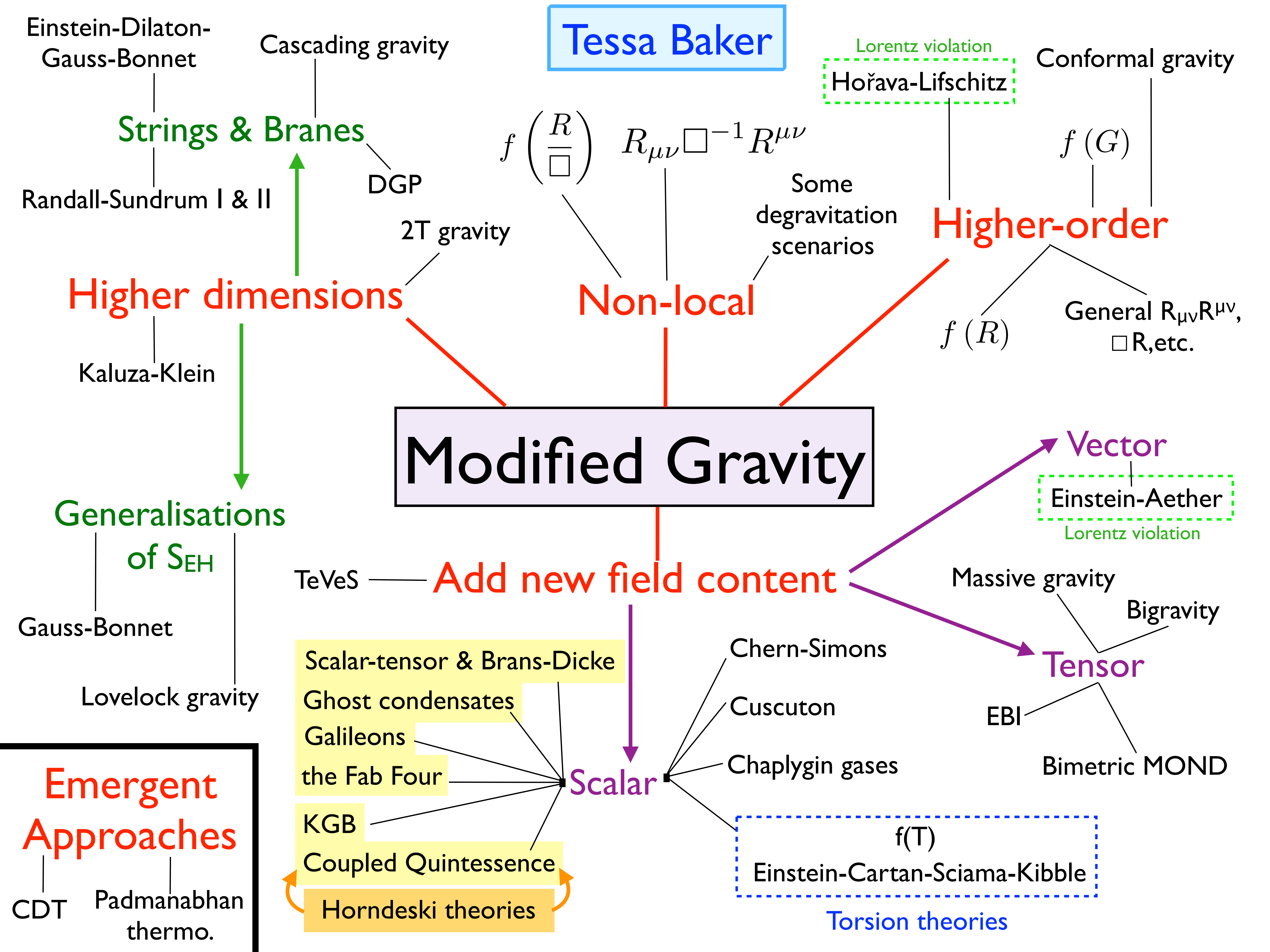
xIST – **Mathematica** routines for linear Scalar Tensor theories.

CoPPer – Cosmological Parameterised Perturbations.

Available from https://github.com/noller/xIST .

Outputs parametrised field equations for LSS + lensing calculations.

- In progress: interface with Einstein-Boltzmann solvers, MCMC, etc.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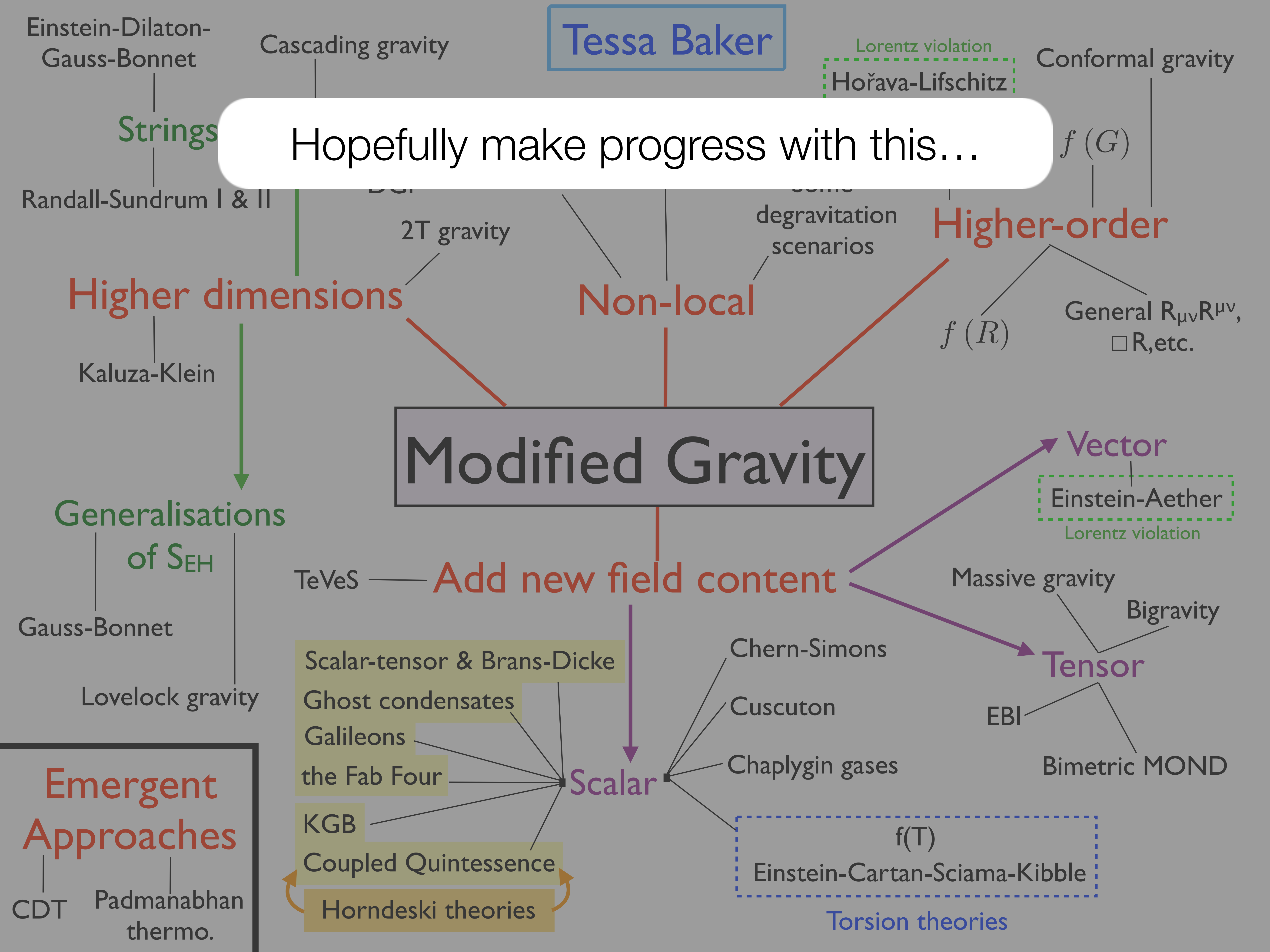
Tessa Baker
Hopefully make progress with this…
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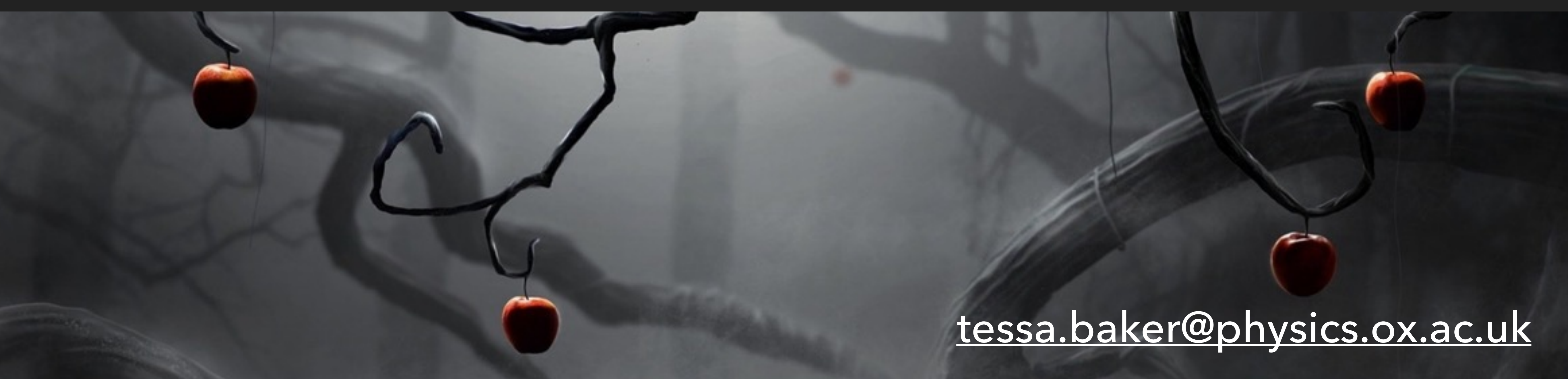
CONCLUSIONS

- Lovelock's theorem and the gravity theory landscape.
- A mathematically rigorous, agnostic parameterisations for testing gravity.
- The current constraint status of these frameworks, and hints of what's to come.



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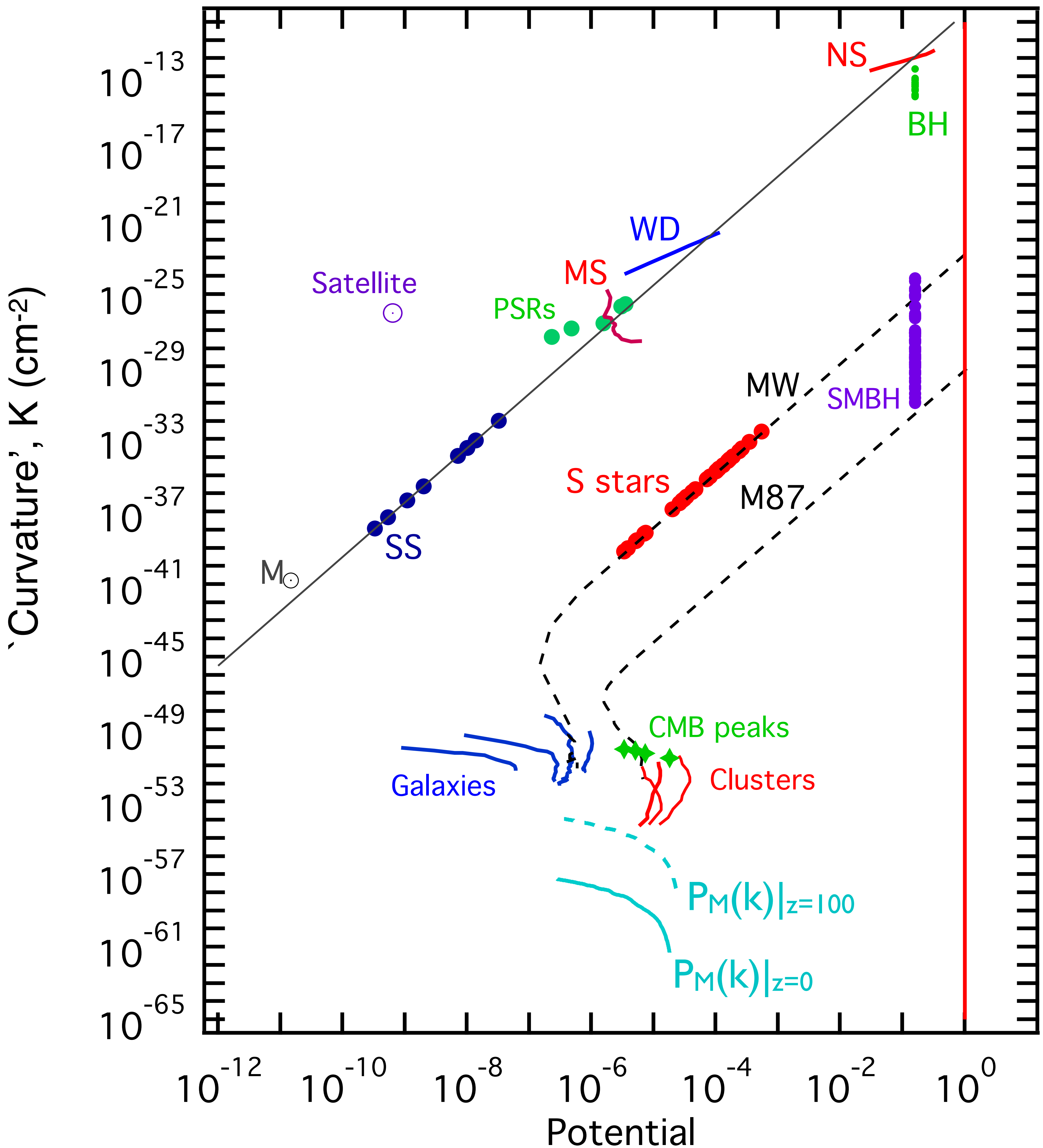
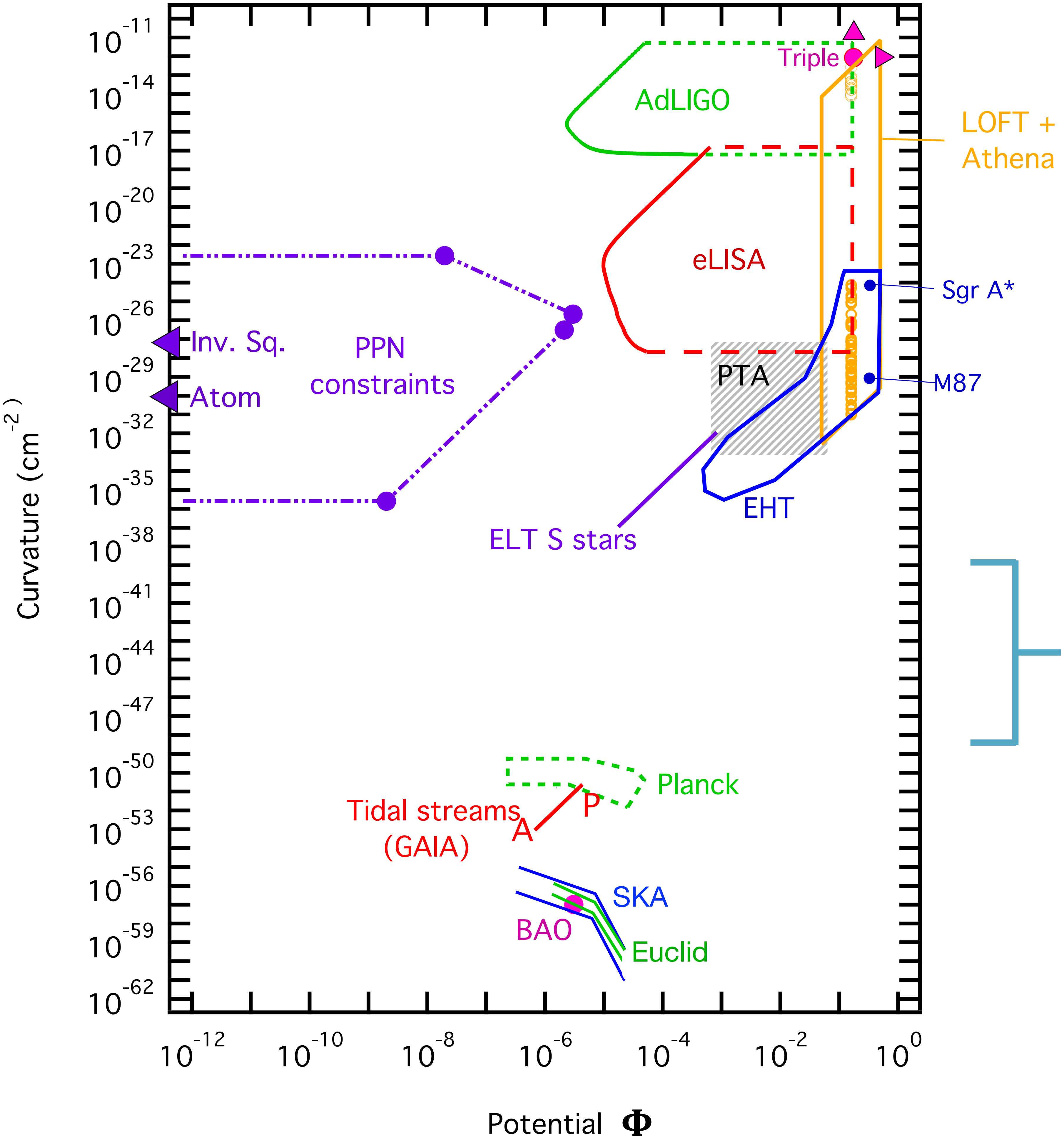
Kretschmann scalar:

$$K = \sqrt{R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}}$$



TB, Psaltis & Skordis (2015).



The experimental version.

Desert?